Common Core State Standard:
6.RP.3.a

Equivalent Ratios

Getting the Idea

Remember, a ratio can be written in simplest form. For example, $\frac{8}{6}$ or 8:6 can be written in simplest form as $\frac{4}{3}$ or 4:3. The ratios $\frac{8}{6}$ and $\frac{4}{3}$ are **equivalent ratios**.

Models can help you tell if two ratios are equivalent or not.

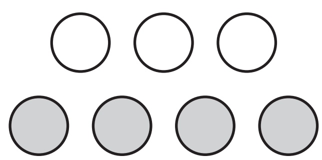
Example 1

Use models to determine if the ratios below are equivalent.

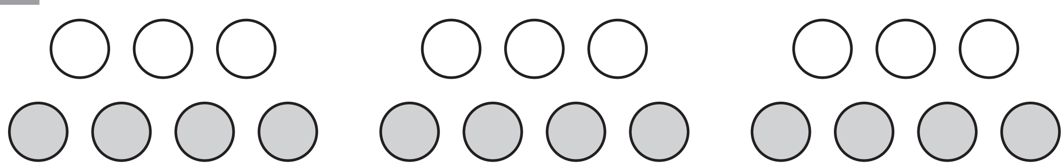
3:4 and 9:12

Strategy **Model each ratio. Then try to separate 9:12 into groups of 3:4.**

Step 1 Model 3:4 using white dots and gray dots.

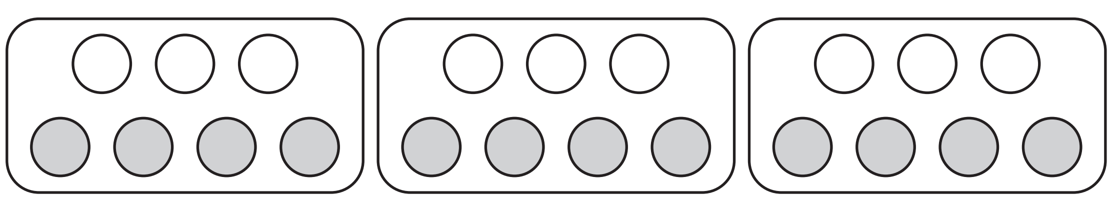


Step 2 Model 9:12 using white dots and gray dots.



Step 3 Try to separate the model for 9:12 into groups of 3:4.

Circle groups of 3 white dots and 4 gray dots.



There are no ungrouped dots, so 3:4 and 9:12 are equivalent ratios.

Solution **The ratios 3:4 and 9:12 are equivalent.**

Another way to tell whether two ratios or rates are equivalent is to **cross multiply**. A rate is a ratio that compares two quantities with different units of measure. To cross multiply, write the ratios as fractions. Then multiply the numerator of each ratio and the denominator of the other ratio. The products are called the **cross products** of the ratios. If the cross products are equal, the ratios are equivalent.

Example 2

There are two photocopy machines in an office. The first machine produces 5 copies in 8 seconds. The second machine produces 16 copies in 24 minutes. Are the two machines making copies at the same rate?

Strategy **Express each rate as a fraction. Then cross multiply to see if the rates are equivalent.**

Step 1 Express each rate as a fraction.

$$\frac{5 \text{ copies}}{8 \text{ minutes}} = \frac{5}{8}$$

$$\frac{16 \text{ copies}}{24 \text{ minutes}} = \frac{16}{24}$$

Step 2 Cross multiply to see if the rates are equivalent.

$$\frac{5}{8} \stackrel{?}{=} \frac{16}{24}$$

$$\frac{5}{8} \stackrel{?}{=} \frac{16}{24}$$

$$5 \times 24 \stackrel{?}{=} 8 \times 16$$

$$120 \neq 128$$

The cross products are not equal, so the ratios are not equivalent.

Solution **The two machines are not making copies at the same rate.**

You can cross multiply to find a missing value in a ratio or rate.

Example 3

Emma is driving at a constant speed. She drives 4 miles in 5 minutes. If she continues to drive at the same speed, how many miles will she drive in 15 minutes?

Strategy **Write two ratios to represent the situation. Cross multiply to find the missing term.**

Step 1 Write two ratios.

The first ratio is: $\frac{4 \text{ miles}}{5 \text{ minutes}}$ or $\frac{4}{5}$.

You want to find the number of miles she will drive in 15 minutes.

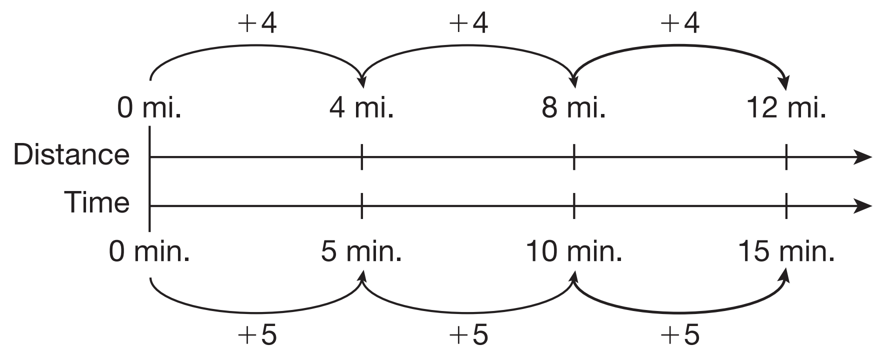
The second ratio is: $\frac{x \text{ miles}}{5 \text{ minutes}}$ or $\frac{x}{15}$.

Step 2 Cross multiply to find the missing term.

$$\frac{4}{5} = \frac{x}{15}$$
$$4 \times 15 = 5 \times x$$
$$60 = 5x$$
$$\frac{60}{5} = \frac{5x}{5}$$
$$12 = x$$

Solution **Emma will drive 12 miles in 15 minutes.**

You could also use a double number line to solve Example 3. The double number line shows the distances and times produced by a constant speed.



The double number line above shows that 4 miles in 5 minutes and 12 miles in 15 minutes are equivalent rates of speed.

You can look for patterns in a table to find equivalent ratios.

Example 4

A baker made this table to show the number of cups of flour and the number of eggs he needs to make a carrot cake recipe.

Carrot Cake Recipe

Cups of Flour (*x*)	2	3	4	5	6
Number of Eggs (*y*)	4	6	8	10	?

How many eggs does the baker need if he uses 6 cups of flour?

Strategy **Look for a rule that relates the number of cups of flour and the number of eggs. Then use that rule to write an equivalent ratio.**

Step 1 Look for a pattern that relates each pair of values in the table.

The first ratio is $\frac{\text{2 cups of flour}}{\text{4 eggs}}$, or $\frac{2}{4}$.

$2 \times \mathbf{2} = 4$, so the rule may be to multiply each x-value by 2.

See if that works for the other ratios, $\frac{3}{6}$, $\frac{4}{8}$, and $\frac{5}{10}$.

$3 \times \mathbf{2} = 6$ ✓

$4 \times \mathbf{2} = 8$ ✓

$5 \times \mathbf{2} = 10$ ✓

Step 2 Write a rule.

The number of eggs (y) equals 2 times the number of cups of flour (x).

The rule is: $y = 2 \times x$ or $y = 2x$.

Step 3 Find the missing value in the table.

If the baker uses 6 cups of flour, $x = 6$.

$y = 2x = 2 \times 6 = 12$

The ratio is 6 cups of flour to 12 eggs, or $\frac{6}{12}$.

The ratio $\frac{6}{12}$ in simplest form equals $\frac{1}{2}$, so those ratios are equivalent.

Solution **If the baker uses 6 cups of flour, he needs 12 eggs.**

You can also plot ordered pairs, (x, y), on a coordinate grid to find equivalent ratios. If the ratios of x:y are equivalent, the points will lie along the same straight line.

Example 5

The table shows the cost of shipping a package for different package weights.

Shipping Costs

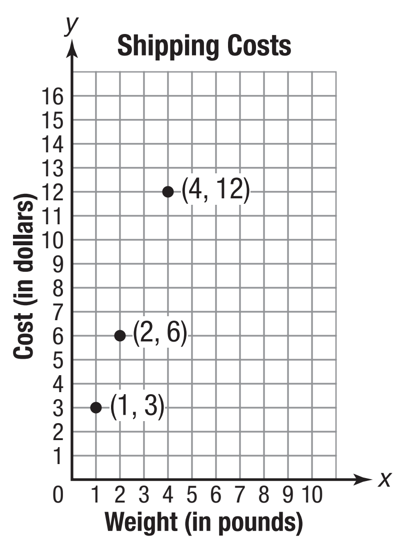
Weight in Pounds (x)	1	2	4
Cost in Dollars (y)	3	6	12

Plot each ordered pair on a coordinate grid. Then use that graph to find two more equivalent rates.

Strategy **Plot the ordered pairs on a coordinate grid. Graph the line for the ordered pairs. Find two more ordered pairs on the line.**

Step 1 Plot points for each pair of values in the table.

Plot (1, 3), (2, 6), and (4, 12) on a coordinate grid.



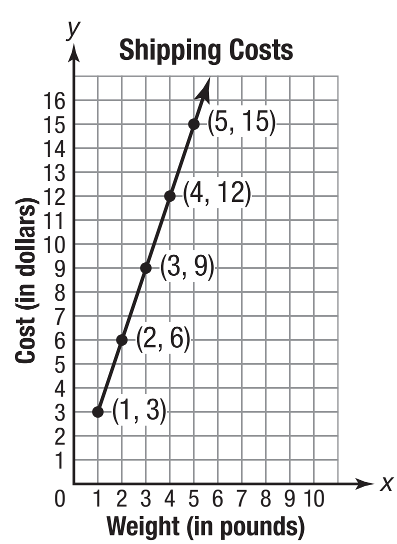
Step 2 Draw a line through the points. Extend the line. Name two other points on the line.

The line through points (1, 3), (2, 6), and (4, 12) shows that the ratios $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{4}{12}$ are equivalent.

The points (3, 9) and (5, 15) are also on the line.

This shows that $\frac{3}{9}$ and $\frac{5}{15}$ are equivalent to the other ratios.

The points represent these rates: 3 pounds for \$9 and 5 pounds for \$15.



Solution **Two additional equivalent rates are 3 pounds for \$9 and 5 pounds for \$15.**

Coached Example

Hudson bought several cans of tennis balls. Each can contained both green and yellow tennis balls. He purchased 10 green tennis balls and 5 yellow tennis balls in all. If each can has 2 green tennis balls in it, how many yellow tennis balls are in each can?

Write two ratios for this problem.

Write the first ratio.

$$\frac{\text{10 green tennis balls}}{\text{\_\_\_\_ yellow tennis balls}} \text{ or } \frac{10}{\_\_\_}$$

You want to find the number of yellow tennis balls in a can of 2 green tennis balls.

Write the second ratio.

$$\frac{\text{2 green tennis balls}}{x \text{ yellow tennis balls}} \text{ or } \frac{2}{x}$$

Set the ratios equal to each other. Cross multiply to find the number of yellow tennis balls in each can.

$$\frac{10}{\_\_\_} = \frac{2}{x}$$

$$10 \times x = \_\_\_\_\_\_ \times 2$$

$$10x = \_\_\_\_\_\_$$

$$\frac{10x}{10} = \_\_\_\_\_\_$$

$$x = \_\_\_\_\_\_$$

Each can contains 2 green tennis balls and ______ yellow tennis ball(s).