Common Core State Standard:
6.NS.4

Factors and Multiples

Getting the Idea

A **multiple** of a number is the product of that number and any of the counting numbers (1, 2, 3, 4, 5, …). For example, some multiples of 7 are shown below.

7, 14, 21, 28, 35, …

The **least common multiple (LCM)** of two numbers is the least number that is a multiple of both numbers.

Example 1

What is the least common multiple (LCM) of 8 and 12?

Strategy **List the multiples of each number. Find the least number that is in both lists.**

Step 1 List the first six multiples of 8.

8, 16, 24, 32, 40, 48

Step 2 List the first six multiples of 12.

12, 24, 36, 48, 60, 72

Step 3 What is the least common multiple?

The numbers 24 and 48 appear in both lists, but 24 is the *least* of those multiples.

Solution **The least common multiple (LCM) of 8 and 12 is 24.**

You can use what you know about the least common multiple to solve a real-world problem.

Example 2

The students in the drama club can be divided into 3 equal groups or 5 equal groups, with no students left over. What is the least number of students that could be in the drama club?

Strategy **Use what you know about the least common multiple.**

Step 1 How can the LCM help you solve this problem?

Since the drama club can be divided into equal groups of 3 or 5, the number of students in the drama club must be a multiple of both 3 and 5.

To find the least possible number of students in the drama club, find the LCM of 3 and 5.

Step 2 List the first six multiples of 3 and 5.

Multiples of 3: 3, 6, 9, 12, 15, 18

Multiples of 5: 5, 10, 15, 20, 25, 30

Step 3 Find the least number that is common to both lists.

The least common multiple is 15.

Solution **The least possible number of students in the drama club is 15.**

The **factors** of a number are the counting numbers that can be multiplied to get that number. (A factor of a number is also any counting number that divides that number evenly.) For example:

$1 \times 10 = 10$, so 1 and 10 are factors of 10.

$2 \times 5 = 10$, so 2 and 5 are also factors of 10.

The number 10 has four factors: 1, 2, 5, and 10.

The greatest common factor (GCF) of two numbers is the greatest number that is a factor of both numbers.

Example 3

What is the greatest common factor (GCF) of 20 and 50?

Strategy **List the factors of each number. Find the greatest number that is in both lists.**

Step 1 List the factors of 20.

1, 2, 4, 5, 10, 20

Step 2 List the factors of 50.

1, 2, 5, 10, 25, 50

Step 3 What is the greatest common factor?

1, 2, 5, and 10 appear in both lists.

The *greatest* of those common factors is 10.

Solution **The greatest common factor (GCF) of 20 and 50 is 10.**

Numbers can be expressed in different ways. For example, the number 10 can be written as the sum $4 + 6$. You can use the **distributive property** to write this sum in another way.

Distributive Property

When you multiply the sum of two numbers by another number, you can multiply each addend by the number, and then add the products. The property also applies to subtraction.

$$a(b + c) = ab + ac$$
$$a(b - c) = ab - ac$$

Using the distributive property: $2(2 + 3) = (2 \times 2) + (2 \times 3)$
$= 4 + 6$
$= 10$

So, you can write 10 as $4 + 6$ and as $2(2 + 3)$. Notice that the numbers in the sum $(2 + 3)$ have no common factors.

Example 4

The number 99 can be expressed as the sum $45 + 54$. Use the distributive property to rewrite that sum as a multiple of a sum whose addends have no common factors.

Strategy **Find the greatest common factor of 45 and 54. Use the distributive property to rewrite the sum.**

Step 1 Find the greatest common factor of 45 and 54.

Factors of 45: 1, 3, 5, 9, 15, and 45

Factors of 54: 1, 2, 3, 6, 9, 18, 27, and 54

The common factors are 1, 3, and 9. So, the GCF is 9.

Step 2 Use the GCF and the distributive property to rewrite the sum.

$45 = 5 \times 9$

$54 = 6 \times 9$

$45 + 54 = 9(5 + 6)$

Step 3 Check that the addends in the sum have no common factors, other than 1.

Factors of 5: 1, 5

Factors of 6: 1, 2, 3, 6

The numbers 5 and 6 have no common factors, other than 1.

Solution **The sum $45 + 54$ can be expressed as $9(5 + 6)$.**

Coached Example

Two P.E. classes are participating in Field Day. One class has 24 students. The other class has 27 students. The P.E. teachers want to divide the students in each class into the largest possible equal groups, with no students left over. If all the groups have the same number of students, how many students are in each group?

If the classes are divided into equal groups, the number of students in each group must be a _____ of 24 and 27.

List the factors of 24: 1, _____, _____, ____, ____, _____, _____, 24

List the factors of 27: 1, _____, _____, 27

What are the common factors of 24 and 27? _____

What is the greatest common factor of 24 and 27? _____

If all the groups have the same number of students, there will be _____ students in each group.