

Lesson 19: Solving Equations

An **equation** is a mathematical statement that has an equal sign, $=$. The equal sign separates two expressions and shows that the expressions are equal. Remember, an expression does not contain an equal sign. Different examples of equations are shown below.

$$12 + 13 = 25 \quad 10 + x = 15 \quad 2x + 4 = 3x + 1$$

Many equations will have a variable on one or both sides. Solving an equation means finding the value for the variable that makes the expressions equal to each other. To solve an equation, you need to isolate the variable (get it alone on one side of the equation). Then you can use inverse operations to find the value of the variable.

You can check a solution by substituting it for the variable in the original equation.

Example

A solution to the following equation is $n = 3$. Is the solution correct?

$$3n + 4 = 13$$

To check the solution, substitute the value for n into the original equation.

$$3n + 4 = 13$$

$$3(3) + 4 \stackrel{?}{=} 13$$

$$9 + 4 \stackrel{?}{=} 13$$

$$13 \stackrel{?}{=} 13$$

Substitute 3 for n .

Multiply 3 by 3.

Add.

Because $13 = 13$, the solution $n = 3$ is correct.

Example

A solution to the following equation is $p = 5$. Is the solution correct?

$$8p = 40$$

To check the solution, substitute the value for p into the original equation.

$$8p = 40$$


$$8(5) \stackrel{?}{=} 40$$

$$40 \stackrel{?}{=} 40$$

Substitute 5 for p .

Multiply 8 by 5.

Because $40 = 40$, the solution $p = 5$ is correct.

 **TIP:** Not every equation has a solution. There may be no solution to an equation, such as $x + 1 = x$.

You can solve equations by using inverse operations to create equivalent equations. If two equations are equivalent, they have the same solution.

Example

Solve the following equation for x . Then check the answer.

$$4 + x = 15$$

Use inverse operations to get the variable alone on one side of the equation.

$$4 + x = 15$$

$$4 + x - 4 = 15 - 4$$

Subtract 4 from both sides.

$$x = 11$$

The solution to the equation is $x = 11$. To check the solution, substitute the value for the variable in the original equation.

$$4 + x = 15$$

$$4 + (11) \stackrel{?}{=} 15$$

Substitute 11 for x .

$$15 \stackrel{?}{=} 15$$

Add.

Because $15 = 15$, the solution is correct. The value of x is 11.

The solution to $x = 11$ is 11. Because $4 + x = 15$ and $x = 11$ are equivalent equations, the solution to $4 + x = 15$ is also 11.

Example

Solve the following equation for y . Then check the answer.

$$8y = 72$$

Use inverse operations to get the variable alone on one side of the equation.

$$8y = 72$$

$$8y \div 8 = 72 \div 8$$

Divide both sides by 8.

$$y = 9$$

The solution to the equation is $y = 9$. To check the solution, substitute the value for the variable in the original equation.

$$8y = 72$$


$$8(9) \stackrel{?}{=} 72$$

Substitute 9 for y .

$$72 \stackrel{?}{=} 72$$

Add.

Because $72 = 72$, the solution is correct. The value of y is 9.

 **TIP:** Addition and subtraction are inverse operations, as are multiplication and division.

You can write an equation from a given scenario. Translate the numbers and operations as you would when writing expressions. However, be sure to include an equal sign to separate the two expressions. The word *is* or *equals* is often an indicator of an equal sign.

▶ Example

Write an equation to represent the following scenario.

A number increased by nine equals ten. Let n = the number.

Because the variable is n , replace “a number” with n . The phrase *increased by* represents addition. The word *equals* should be represented by an equal sign.

Now you can write the equation.

A number increased by nine equals ten

$$\begin{array}{cccccc} \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ n & & + & & 9 & & = & & 10 \end{array}$$

The equation that represents the scenario is $n + 9 = 10$.

▶ Example

Write an equation to represent the following scenario.

The product of a number and five is fifteen. Let x = the number.

Replace “a number” with x . The word *product* represents multiplication. The word *is* should be represented by an equal sign.

Remember that when a variable is multiplied by a number, you do not need the multiplication symbol. The number becomes the coefficient of the variable. Now write the equation.

The product of a number and five is fifteen

$$\begin{array}{ccc} \underbrace{\hspace{10em}} & & \downarrow \quad \downarrow \\ & & 5x \quad = \quad 15 \end{array}$$

The equation that represents the scenario is $5x = 15$.

